PHYSICS NYB-10/11 Winter 2007

Lecture 10: Electrical power

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Review

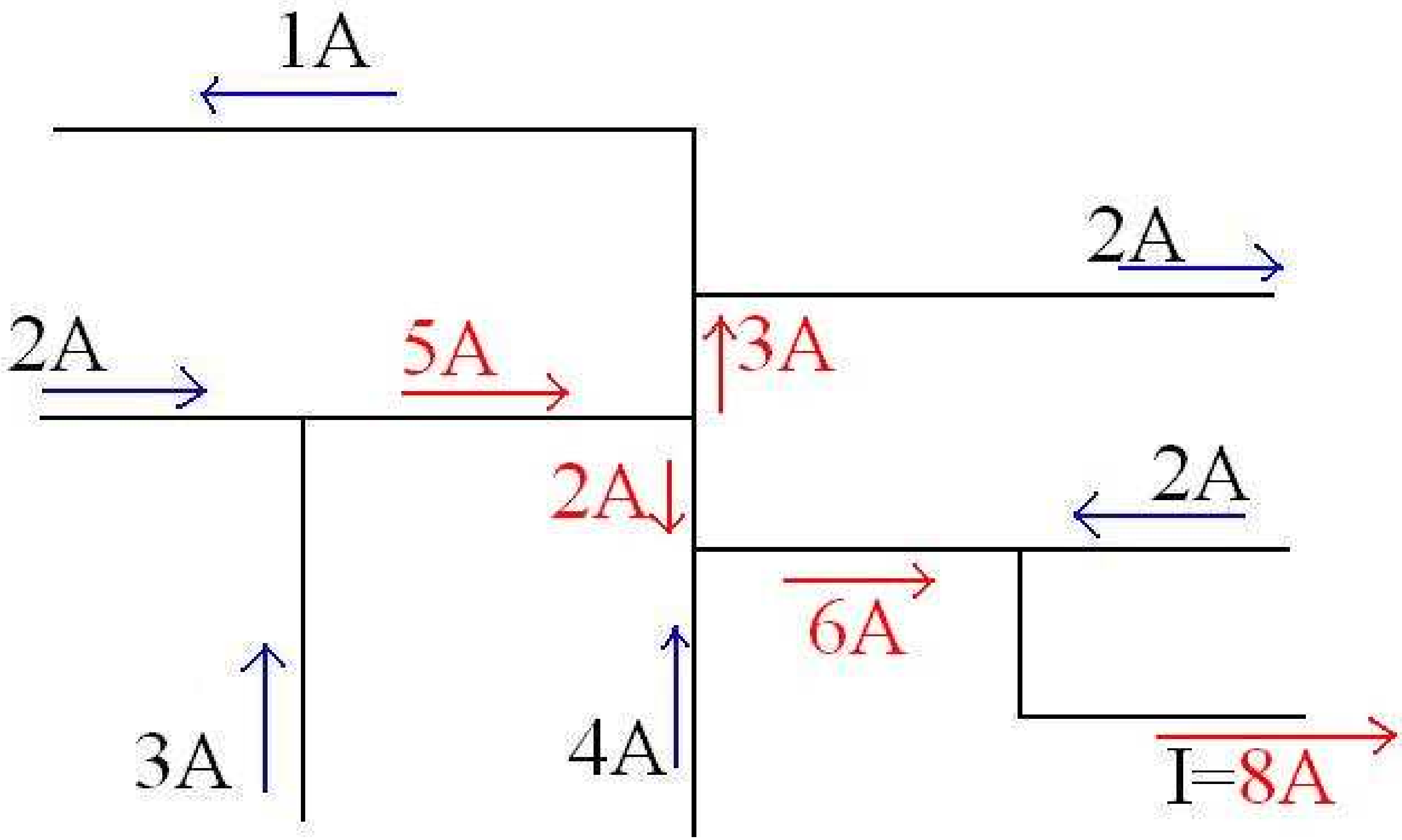
- Current is the rate at which charge flows $I = \dfrac{dQ}{dt}$
- Microscopically, the current in a wire is $I = nqAv_d$, where nq is the volume charge density, A is the area and v_d is the *drift speed* of the charge carriers
- Drift speed is typically very low, though *signals* travel fast
- $\vec{v}_d = \dfrac{q\vec{E}}{m}\tau$ where τ is the average time between collisions
- Current is *conserved*. What goes into a given point must come out of the same point.

Review

- Current *density* is the current per unit area, $J = \dfrac{I}{A}$
- Ohm's law: the current density in a given material is proportional to the electric field $\vec{J} = \sigma\vec{E} = \dfrac{1}{\rho}\vec{E}$
- σ is the *conductivity* and $\rho = \dfrac{1}{\sigma}$ the *resistivity*. They are properties of the material.
- Ohm's law implies that $I = \dfrac{\Delta V}{R}$ where R is the resistance
- For a wire of length l and area A, the resistance is $R = \rho\dfrac{l}{A}$

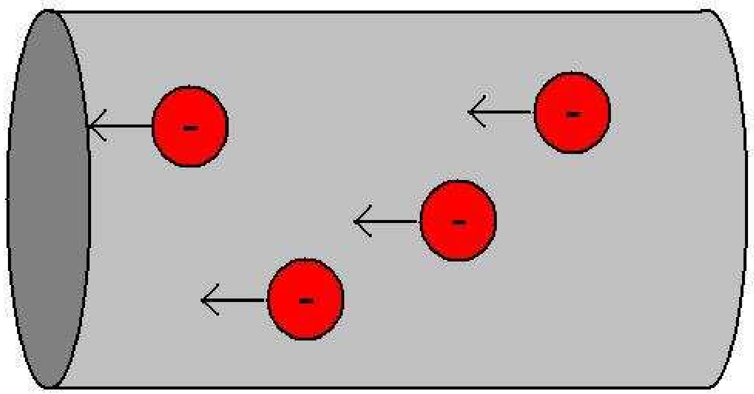
Examples

What is the magnitude and direction of the current I in the portion of circuit pictured here?



Examples

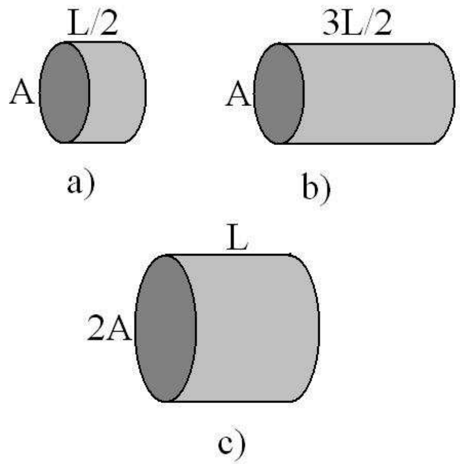
What are the directions of I, $\vec{J}$, $\vec{v}_d$ and $\vec{E}$ in the situation pictured below?



Charges are moving toward the left, so $\vec{v}_d$ points to the left. The charges are negative, so the current, current density and electric field are all toward the right.

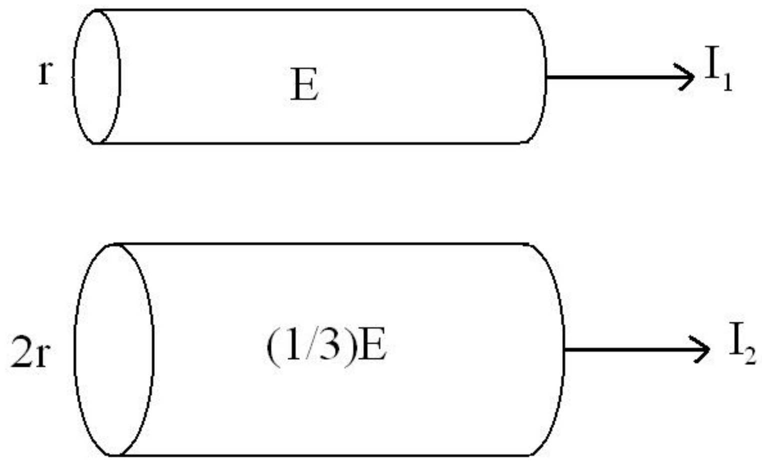
Examples

Order the three wires (made of the same material) below in terms of the current flowing through them if the same potential difference ΔV is applied to each.



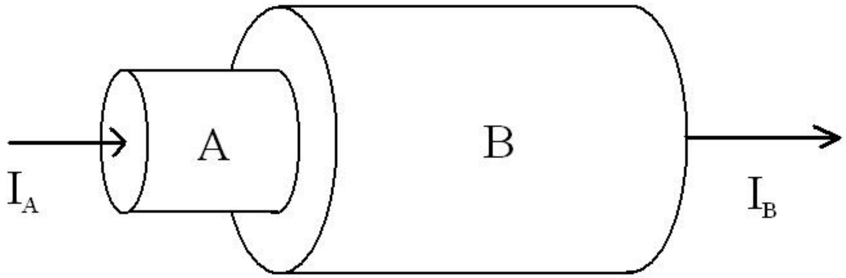
We use $R = \rho \frac{l}{A}$ and $I = \frac{\Delta V}{R}$ to find that the current is greatest (and the same) for a) and c), where $R = \rho \frac{L}{2A}$. The current will be less in wire b), since $R = \rho \frac{3L}{2A}$.

Examples



The two wires above are made of the same material. Compare I_1 and I_2. Compare J_1 and J_2.
We know $I = nqAv_d$, and we saw that $v_d \sim E$. The charge density nq depends on the material, which is the same for both wires. The area is proportional to the square of the radius, so all in all $I \sim r^2E$, so $I_1/I_2 = (r_1^2E_1)/(r_2^2E_2) = 3/4$. The current density $J = \sigma E$, so the ratio $J_1/J_2 = E_1/E_2 = 3$.

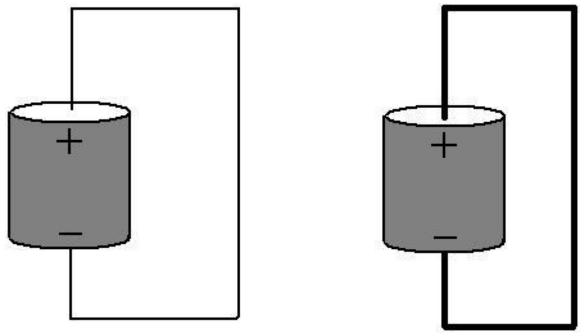
Examples



A wire consists of two segments of different diameters. Compare I_A and I_B as well as J_A and J_B.

Here, $I_A = I_B$, or else charge would be accumulating in one of the segments. Since $J = I/A$, the ratio J_A/J_B is equal to the ratio A_B/A_A, or r_B^2/r_A^2.

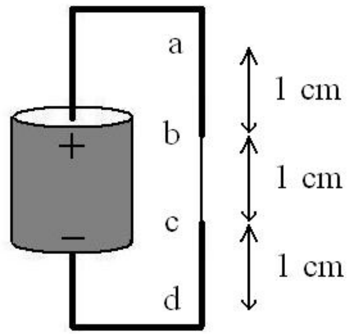
Examples



Two wires of equal length and indentical metals but different diameters are connected to identical batteries. Compare currents I_1 and I_2.

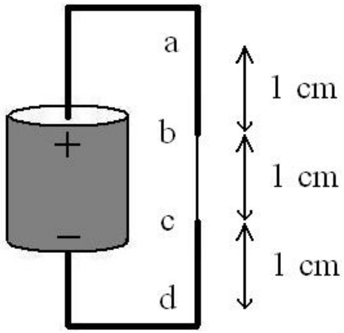
Here, we use $I = \Delta V/R$ along with the fact that $R = \rho l/A$. Since both wires are the same length and same metal, ρ and l are the same. The potential difference is also the same because of the indentical batteries, so $I_1/I_2 = A_1/A_2$ and the current is bigger in the fat wire than in the thin one.

Examples



A wire connected to a battery has sections of different diameters.
a) Compare the currents I in the top, middle and bottom segments.
b) Compare the current densities J in the top, middle and bottom segments.
c) Compare the potential differences ΔV_{ab}, ΔV_{bc} and ΔV_{cd}.

Examples



The current is the same in all segments. Therefore, the current density is bigger in the smaller wire. Current density is $J = \sigma E$, and σ is the same for all three wires (made of the same metal), so E is larger in the smaller wire. Therefore, since E is $\Delta V/\Delta x$, the potential difference over a one centimeter distance is bigger for the thin wire where E is bigger.

Examples

Suppose that the current through a conductor decreases exponentially with time according to the equation $I(t) = I_0 e^{-t/\tau}$ where I_0 is the initial current (at $t = 0$), and τ is a constant having dimensions of time. Consider a fixed observation point within the conductor. (a) How much charge passes this point between $t = 0$ and $t = \tau$? (b) How much charge passes this point between $t = 0$ and $t = 10\tau$? (c) How much charge passes this point between $t = 0$ and $t = \infty$?

Examples

By definition, the current is the rate at which charge is flowing, $I(t) = \frac{dQ}{dt}$. This means we can write $I(t)dt = dQ$ or $\int_0^T I(t)dt = \int_{Q_i}^{Q_f} dQ$ so that

$$\begin{aligned}\int_{Q_i}^{Q_f} dQ = Q_f - Q_i = \Delta Q &= \int_0^T I_0 e^{-t/\tau} dt \\ &= -I_0 \tau e^{-t/\tau}\Big|_0^T \\ \Delta Q &= I_0\tau(1 - e^{-T/\tau})\end{aligned}$$

Plugging in $T = \tau$, $T = 10\tau$ and $T = \infty$, we find $\Delta Q = I_0\tau(0.632)$, $\Delta Q = I_0\tau(0.999\ 95)$ and $\Delta Q = I_0\tau$ respectively.

Examples

The electron beam emerging from a certain high-energy electron accelerator has a circular cross section of radius 1.00 mm. (a) The beam current is $8.00\ \mu$A. Find the current density in the beam, assuming that it is uniform throughout. (b) The speed of the electrons is so close to the speed of light that their speed can be taken as $c = 3.00 \times 10^8$ m/s with negligible error. Find the electron density in the beam. (c) How long does it take for Avogadro's number of electrons to emerge from the accelerator?

Examples

Current density is $J = \frac{I}{A} = \frac{8\times10^{-6}}{\pi(10^{-3})^2} = 2.55$ A/m^2. We also know that $J = nqv_d$, so $n = \frac{J}{ev_d} = 5.31 \times 10^{10}$ electrons/m^3. Finally, if we have 8 μC/s passing through, and each electron corresponds to 1.6×10^{-19} C, we have $\frac{8\times10^{-6}}{1.6\times10^{-19}} = 5 \times 10^{13}$ electrons per second. It will therefore take $\frac{6.02\times10^{23}}{5\times10^{13}} = 1.2 \times 10^{10}$ seconds, or about 380 years.

Examples

Gold is the most ductile of all metals. For example, one gram of gold can be drawn into a wire 2.40 km long. What is the resistance of such a wire at 20^o C?

Examples

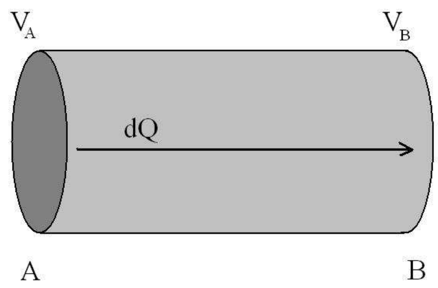
We want to find the resistance, and we know that $R = \rho\frac{l}{A}$. We already know l, and we can look in a table and find that gold at 20^{o}C has a resistivity $\rho = 2.44 \times 10^{-8}$ Ω·m. However, we need to find A. We can do this by noting that the volume density $\rho = 19.3 \times 10^3$ kg/m^3. (Note that we are forced to use ρ in both its uses here...) Therefore the volume is $V = \frac{m}{\rho} = 5.18 \times 10^{-8}$ m^3. We know that the length is 2.4 km, so the area can be found from $V = lA$, giving us $A = 2.16 \times 10^{-11}$ m^2. We are now ready to plug this into the equation for R, giving us $R = 2.71 \times 10^6$ Ω.

Power

- When charges move in a material, they collide with particles in the material
- These collisions transfer energy to the particles, and thus to the material
- This means energy is being *taken away from the current*
- *Power* is the rate at which this energy transfer takes place
- Let's find an expression for electrical power

Power

Imagine you have a wire with resistance R, and you apply a potential difference ΔV between its ends.



Imagine an amount of charge dQ moves from A to B. The energy transfer to the material is $dE = dQ\Delta V$, but we know $I = \dfrac{dQ}{dt}$ so $dE = Idt\Delta V$, or $\dfrac{dE}{dt} = P = I\Delta V$.

$$\boxed{P = I\Delta V = RI^2 = \frac{\Delta V^2}{R}}$$

Power

$$\boxed{P = I\Delta V = RI^2 = \frac{\Delta V^2}{R}}$$

Things to note:

- There is a potential difference and a resistance in the wires of a circuit.
- The resistance in wires is tiny compared to the resistance in resistors, and similarly for the potential difference, which is why we always (justifiably!) neglect them for circuits
- The potential energy that the battery gives the charges in a circuit is transferred into other types of energy by a resistor
- This is what a resistor is *supposed* to do; energy transfer in a resistor <u>is not a source of error</u>!!!

Power

Things to note:

- The power rating on an electrical device assumes you are plugging it alone on to a 120 V potential difference.
- The resistance in an electrical device is what is fixed.
- *The power drawn by electrical devices thus depends on where we plug them!*

Power

Using $P = \Delta V I = \frac{\Delta V^2}{R}$, we find $I = 5$ A, and $R = 24\ \Omega$ when the toaster is plugged in a 120 V source. If we plug it to a 240 V source, we must first realise that the resistance in the toaster remains the same. So we can now use $P = \frac{\Delta V^2}{R}$ to obtain $P = 2400 W$, as well as now $P = \Delta V I$ to obtain 10 A.

Example

We estimate that 270 million plug-in electric clocks are in the United States, approximately one clock for each person. The clocks convert energy at the average rate 2.50 W. To supply this energy, how many metric tons of coal are burned *per hour* in coal-fired electric generating plants that are, on average, 25.0% efficient? The heat of combustion for coal is 33.0 MJ/kg.

Examples

2.5 Watts is 2.5 J/s. There are 270 million alarm clocks, so $270 \times 10^6 \times 2.5 \times 3600 = 2.43 \times 10^{12}$ J/hour are needed to power these alarm clocks. Each metric ton of coal provides you with $0.25 \times 33 \times 10^6 \times 1000 = 8.25 \times 10^9$ J of energy. This means you have to burn $\frac{2.43\times10^{12}}{8.25\times10^9} = 295$ metric tons of coal every hour simply to power alarm clocks!

Assignment 4

- Chapter 27, problems 2, 6, 11, 17, 24, 26, 42, 44, 56

What to read for next lecture

- 28.2